1. INTRODUCTION

When a cone is cut by a plane in different ways, different curves are obtained. These curves are called conic sections. Depending on the position of the cutting plane, the section may be a circle, ellipse, parabola, or hyperbola. This lesson mainly studies the ellipse, parabola, and hyperbola, their definitions, standard equations, and important properties.

2. CONIC SECTION

A conic section is the locus of a point P which moves so that its distance from a fixed point is always in a constant ratio to its perpendicular distance from a fixed straight line.

$$PS / PM = e$$

- Focus: The fixed point is called the focus.
- Directrix: The fixed straight line is called the directrix.
- Axis: The straight line passing through the focus and perpendicular to the directrix is called the axis.
- Eccentricity: The constant ratio is called the eccentricity and is denoted by e.
- Latus rectum: The double ordinate passing through the focus and parallel to the directrix is called the latus rectum.

Value of e	Conic obtained
$e < 1$	Ellipse
$e = 1$	Parabola
$e > 1$	Hyperbola

3. ELLIPSE

An ellipse is the locus of a point which moves in a plane such that its distance from a fixed point bears a constant ratio, less than unity, to its distance from a fixed line.

$$e < 1$$

3.1 Standard Equation

For an ellipse with centre at the origin and major axis along the x-axis:

$$x^2/a^2 + y^2/b^2 = 1$$

Here, a is the semi-major axis and b is the semi-minor axis. The source uses the relation:

$$b^2 = a^2(1 - e^2)$$

$$e = \sqrt{1 - b^2/a^2}$$

3.2 Important Properties

Property	Result
Centre	(0, 0)
Vertices	$(\pm a, 0)$
Major axis	Length = 2a
Minor axis	Length = 2b
Foci	$(\pm ae, 0)$
Directrices	$x = \pm a/e$
Length of latus rectum	$2b^2/a$

3.3 Worked Example

Question: Find the eccentricity, coordinates of the foci, and lengths of the axes of the ellipse $3x^2 + 4y^2 = 12$.
Step 1: Divide by 12.

$$x^2/4 + y^2/3 = 1$$

Therefore, $a^2 = 4$ and $b^2 = 3$. Hence $a = 2$ and $b = \sqrt{3}$.
Step 2: Eccentricity $e = \sqrt{(1 - 3/4)} = 1/2$
Step 3: Foci : $(\pm ae, 0) = (\pm 1, 0)$
Step 4: Lengths of axes
Major axis = $2a = 4$; Minor axis = $2b = 2\sqrt{3}$

4. PARABOLA

A parabola is the locus of a point which moves in a plane so that its distance from a fixed point is equal to its distance from a fixed line.

$$PS = PM \text{ and } e = 1$$

4.1 Standard Equation

The standard equation of a parabola opening towards the right is: $y^2 = 4ax$

Property	Result
Vertex	$(0, 0)$
Focus	$(a, 0)$
Axis	$y = 0$
Directrix	$x = -a$
Length of latus rectum	$4a$

4.2 Other Forms of the Parabola

Equation	Opening	Focus	Directrix
$y^2 = 4ax$	Right	$(a, 0)$	$x = -a$
$y^2 = -4ax$	Left	$(-a, 0)$	$x = a$
$x^2 = 4ay$	Upward	$(0, a)$	$y = -a$
$x^2 = -4ay$	Downward	$(0, -a)$	$y = a$

4.3 Method for Finding an Equation from Focus and Directrix

Let $P(x, y)$ be any point on the parabola. Use the defining property: Distance of P from focus = Perpendicular distance of P from directrix

5. HYPERBOLA

A hyperbola is the locus of a point which moves in a plane such that the ratio of its distance from a fixed point to its distance from a fixed straight line is greater than unity.

$$e > 1$$

5.1 Standard Equation with Transverse Axis Along the x-axis

$$x^2/a^2 - y^2/b^2 = 1$$
$$b^2 = a^2(e^2 - 1)$$
$$e = \sqrt{(a^2 + b^2)} / a$$

Property	Result
Centre	$(0, 0)$
Vertices	$(\pm a, 0)$
Foci	$(\pm ae, 0)$
Transverse axis	Along x-axis; length = $2a$
Conjugate axis	Along y-axis; length = $2b$
Directrices	$x = \pm a/e$
Length of latus rectum	$2b^2/a$

5.2 Hyperbola with Transverse Axis Along the y-axis

$$y^2/b^2 - x^2/a^2 = 1$$

Property	Result
Centre	(0, 0)
Vertices	(0, ±b)
Foci	(0, ±be)
Transverse axis	Along y-axis; length = 2b
Conjugate axis	Along x-axis; length = 2a
Directrices	$y = \pm b/e$
Length of latus rectum	$2a^2/b$

5.3 Rectangular Hyperbola

If the length of the transverse axis is equal to the length of the conjugate axis, the hyperbola is called a rectangular hyperbola. Thus, a = b.

$$x^2 - y^2 = a^2$$

For a rectangular hyperbola: $e = \sqrt{2}$

5.4 Worked Example

Question: For the hyperbola $x^2/16 - y^2/9 = 1$, find the eccentricity, foci, vertices, directrices, lengths of the axes, and latus rectum.

Here $a^2 = 16$ and $b^2 = 9$. Therefore, a = 4 and b = 3.

$$e = \sqrt{(16 + 9)}/4 = 5/4$$

- Foci = (±ae, 0) = (±5, 0)
- Vertices = (±4, 0)
- Directrices: $x = \pm a/e = \pm 16/5$
- Length of transverse axis = 2a = 8 units
- Length of conjugate axis = 2b = 6 units
- Length of latus rectum = $2b^2/a = 9/2$ units

COMPLETE FORMULA SUMMARY

Conic	Condition	Standard equation	Key result
Ellipse	$e < 1$	$x^2/a^2 + y^2/b^2 = 1$	Foci (±ae,0); L.R. = $2b^2/a$
Parabola	$e = 1$	$y^2 = 4ax$	Focus (a,0); L.R. = 4a
Hyperbola	$e > 1$	$x^2/a^2 - y^2/b^2 = 1$	Foci (±ae,0); L.R. = $2b^2/a$